

Elmer capabilities in Electromagnetism

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Electromagnetics tools in Elmer

- Electrostatic potential
- Static electric force
- Static current conduction
- Magnetostatics
- Induction equation for magnetohydrodynamics

Electrostatic potential – equations

- The potential may be solved from the Poisson equation

$$-\nabla \cdot \varepsilon \nabla \phi = \rho. \quad (1)$$

- The electric field

$$\vec{E} = -\nabla \phi. \quad (2)$$

- Energy density of the field

$$e = \frac{1}{2} \vec{E} \cdot \vec{D} = \frac{1}{2} \varepsilon (\nabla \phi)^2. \quad (3)$$

- The capacitance

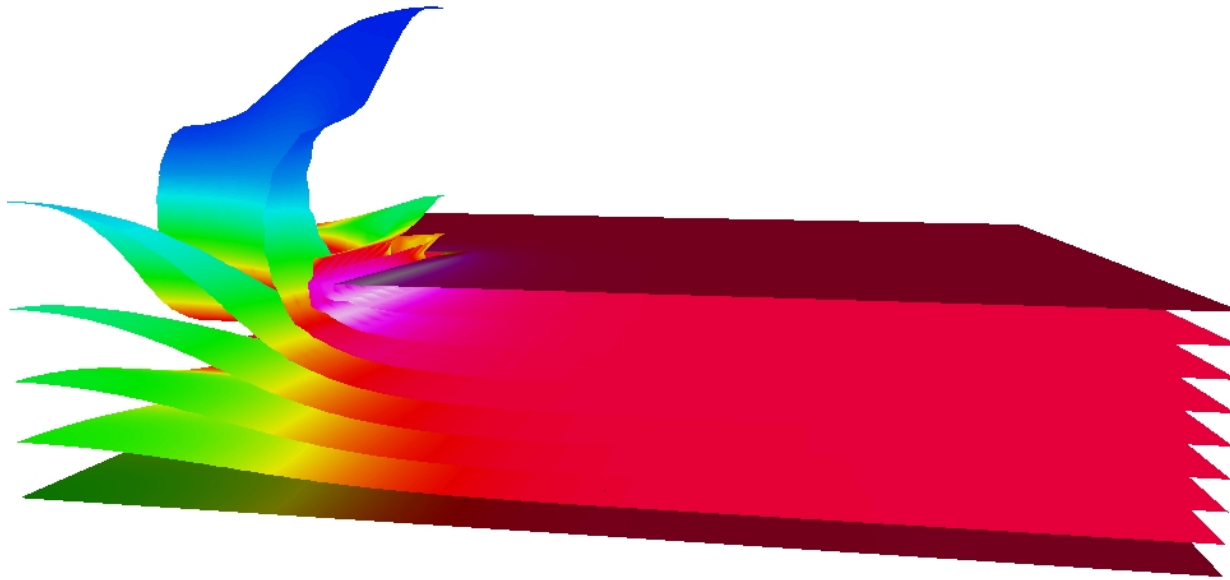
$$C = \frac{1}{\Phi^2} \int_{\Omega} \varepsilon (\nabla \phi)^2 d\Omega. \quad (4)$$

Electrostatic potential – usage

- Solver: StatElecSolve
- Models Manual chapter: Electrostatics
- Tutorials: Electrostatics and Computation of a capacitance matrix
- Material parameter: Relative Permittivity
- Possible also to compute capacitance matrix for a number of conductors

Electrostatic potential – example

- Electric potential and electric field between mismatched capacitor plates



The isosurfaces of the electrostatic potential colored with electric field magnitude.

Static electric force – equations

- Computes the electric force that an electrostatic field causes on a surface
- The force is computed from the Maxwell stress tensor

$$\vec{F} = \int_S \vec{T} \cdot d\vec{S}. \quad (5)$$

- The components of the Maxwell stress tensor for linear medium are

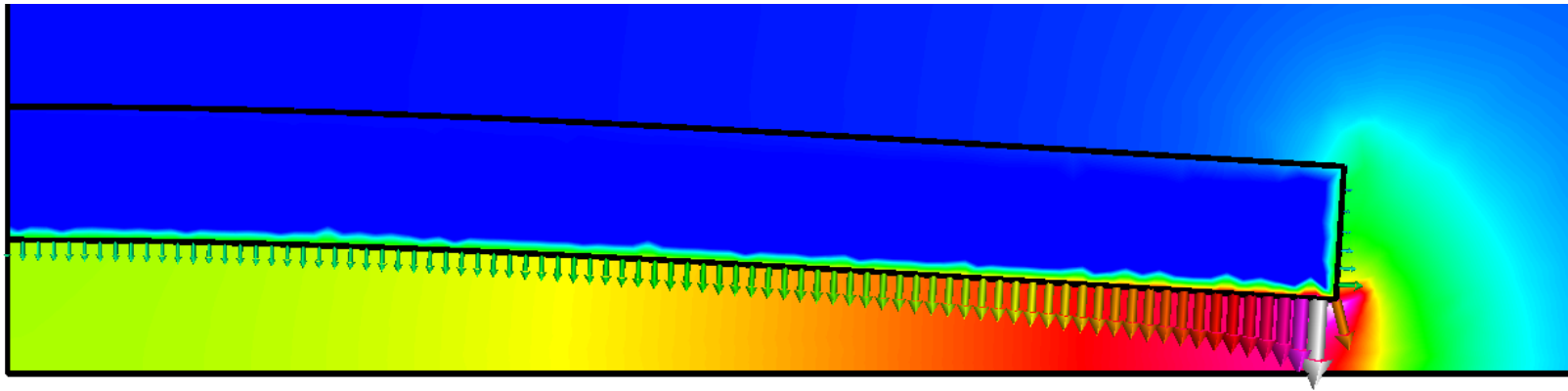
$$T_{ij} = -D_i E_j + \frac{1}{2} \delta_{ij} \vec{D} \cdot \vec{E}. \quad (6)$$

Static electric force – usage

- Solver: `ElectricForce`
- Models Manual chapter: `Static electric force`
- Tutorials: `Electrostatics`
- Material parameter: `Relative Permittivity`
- Give `Calculate Electric Force = Logical True` on all boundaries where electric force should be computed

Static electric force – example

- Electric force used to deform an elastic beam



Electric force (arrows) and the magnitude of the electric field (colors)

Static current conduction – equations

- This module solves the volume current distribution inside a conductor
- The conducting potential is found from Poisson equation

$$\nabla \cdot \sigma \nabla \phi = \frac{\partial \rho}{\partial t}. \quad (7)$$

- The volume current density

$$\vec{J} = -\sigma \nabla \phi. \quad (8)$$

- The electric power loss density which is turned into heat (Joule heating)

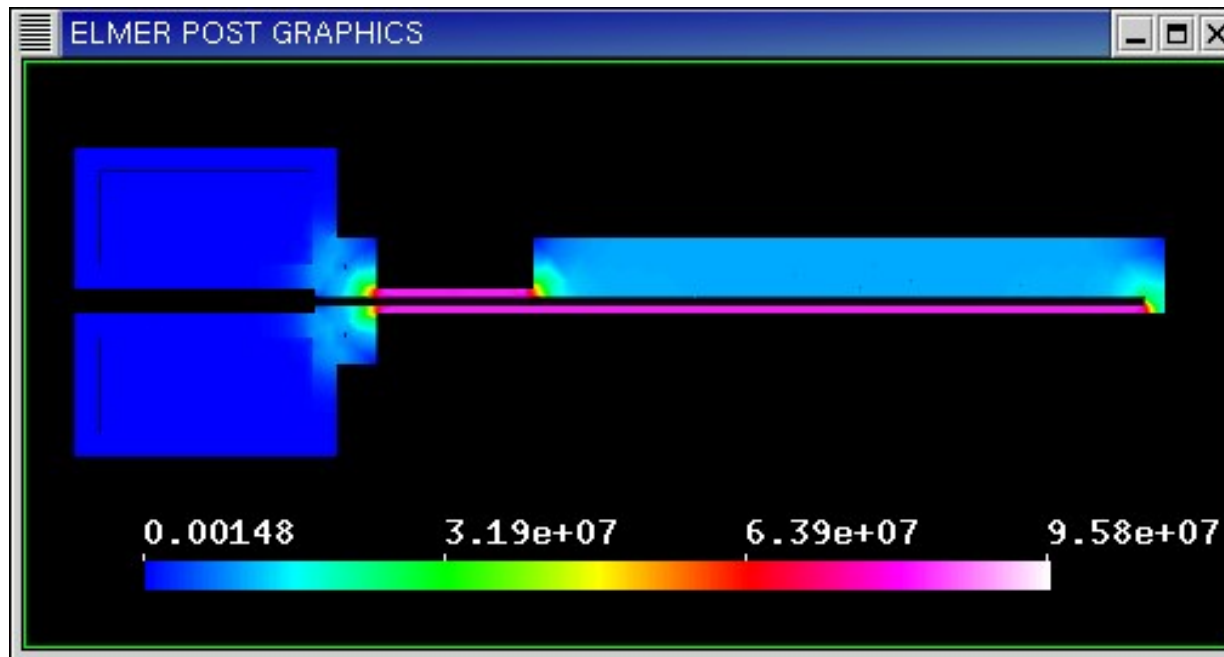
$$h = \nabla \phi \cdot \sigma \nabla \phi. \quad (9)$$

Static current conduction – usage

- Solver: `StatCurrentSolve`
- Models Manual chapter: `Static current conduction`
- Tutorials: `Thermal actuator driven with electrostatic currents`
- Material parameter: `Electric Conductivity`
- This solver is often used coupled with heat equation

Static current conduction – example

- Joule heating used to deform a micromechanical actuator



The volume current distribution in the actuator

Magnetostatics – equations

- Computes the static magnetic field caused by an electromagnet with given current distribution
- The magnetic vector potential is solved from

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \vec{A} \right) = \vec{j}. \quad (10)$$

- Axisymmetric vector potential of a sinusoidally varying current distribution

$$\nabla \times \left(\frac{1}{\mu} \nabla \times A_\phi \vec{e}_\phi \right) + i\omega\sigma A_\phi \vec{e}_\phi = j_\phi \vec{e}_\phi. \quad (11)$$

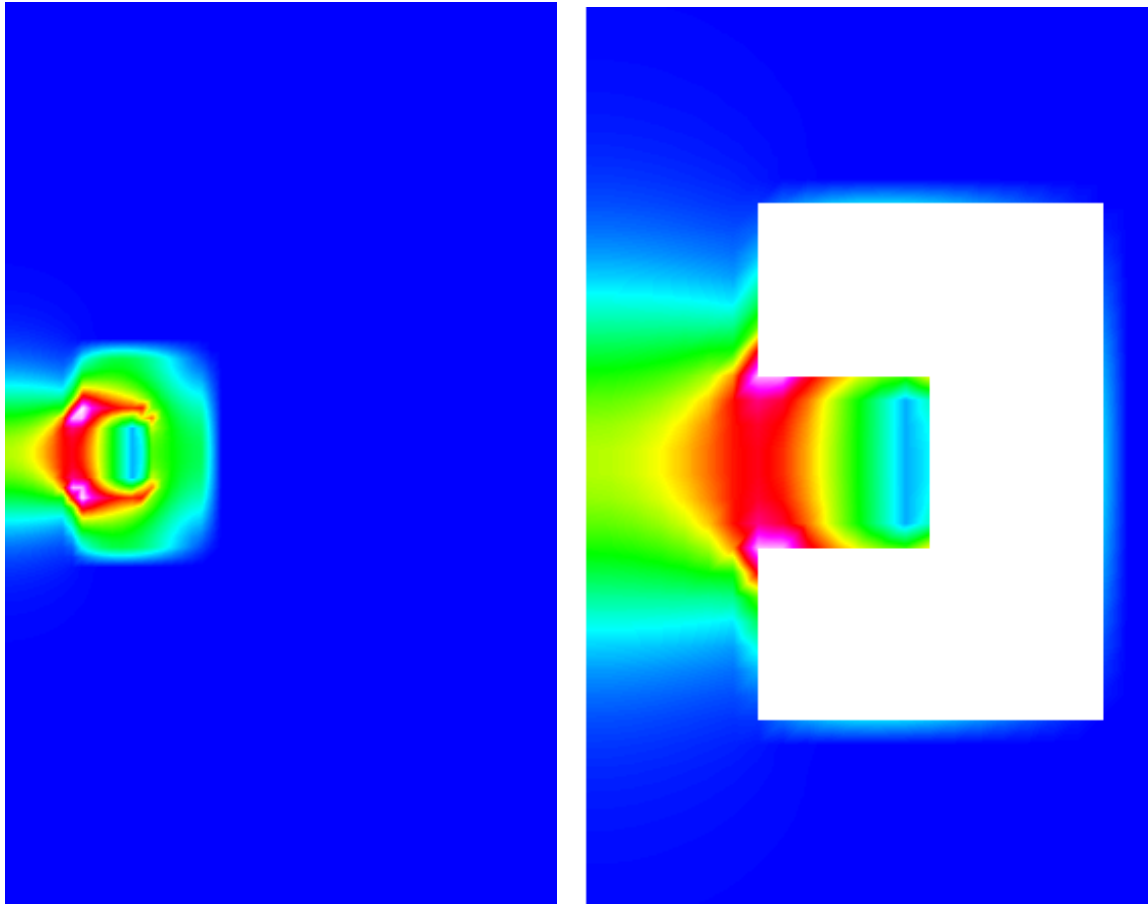
- Heat generation in the sinusoidal case

$$h = \frac{1}{2} \sigma \omega^2 |\vec{A}_0|^2. \quad (12)$$

Magnetostatics – usage

- Solver: StatMagSolve
- Models Manual chapter: Magnetostatics
- Tutorials: Induction heating of a graphite crucible
- Material parameters: Magnetic Permeability and Electric Conductivity (harmonic case)
- May be used to compute external magnetic field for induction equation or to compute field for induction heating

Magnetostatics – example



The magnitude of the magnetic flux vector induced by current driven magnet in axisymmetric geometry, the whole computational domain (left) and a zoom with the magnet removed from the picture (left)

Induction – equations

- The magnetic induction equation describes interaction of a conducting liquid with applied and induced magnetic fields
- The magnetic induction equation is given by

$$\frac{\partial \vec{B}}{\partial t} + \frac{1}{\sigma \mu} \nabla \times \nabla \times \vec{B} - \nabla \times (\vec{v} \times \vec{B}) = 0. \quad (13)$$

- The magnetic field is further divided into induced and external parts $\vec{B} = \vec{B}^e + \vec{B}^i$
- The magnetic force influencing the fluid flow

$$\vec{f}_m = \vec{j} \times \vec{B}, \quad (14)$$

Induction equation – usage

- Solver: `MagneticSolve`
- Models Manual chapter: `Induction equation`
- Tutorials: none
- Material parameters: `Magnetic Permeability` and `Electric Conductivity`
- Used in computing magnetohydrodynamics

Still missing...

- Efficient BEM solver is missing in Elmer
- There is a standard BEM implementation but this is too slow for real problems
- The solution would be a Fast Multipole Method type of acceleration, for example