Full Stokes Modelling of Glaciers and Ice Sheets

Introduction

- Shallow Ice Approximation (SIA):
  - state-of-the-art
  - highly economical
  - numerically easy

- SIA not valid if $H \sim L$

- Full Stokes:
  - Higher horizontal resolution feasible/necessary
  - Increase in PDE for Stokes: $1 \text{ (2D)} \rightarrow 4 \text{ (3D)}$
  - On ice sheet scale $\rightarrow$ Supercomputing
Gorshkov crater glacier

- Compressible Stokes equations (all stresses considered)
- Firm model by Meyssonnier and Gagliardini
- Heat transfer accounting for upper limit of pressure melting point \( (T \leq T_m) \)
  - variational inequality
- Non-linear, thermo-mechanically coupled rheology
- Discontinuous Galerkin method for age/depth relation

Greenland Ice Sheet

- **Full-Stokes (FS):** Finite Element Code ELMER
- **Shallow Ice Approximation (SIA):** SICOPOLIS
- Both models apply Glen’s flow-law and are thermo-mechanically coupled
- Run on 10 x 10 km mesh (Elmer: 210k elements)
- 4 – 32 partitions

Balance velocity distribution


A new ice thickness and bedrock elevation data set for Greenland, part I

JGR, 106 (D24) 33
How to solve FS on an ice sheet

- Finite Element Method (FEM)
- Domain decomposition (MPI for inter-process communication)
- Krylov subspace method for linear solver
- Preconditioner: ILU decomposition
- Fixed point iteration for non-linear problems

Elmer/Ice

Issues:
- Highly elongated elements: $\varepsilon \sim 1/100$
  - SUPG stabilization does not work very well
  - Prohibits better vertical resolution (Elmer 10, SICOPOLIS 81 layers)
- Sensitive to viscosity variations in low shear regions (singularity!)

Strategies:
- Residual free bubbles stabilization
- Smoothing of viscosity factor (from nodes to element and back)
- Cut-off value for shear rate at $10^{-3}$ a$^{-1}$

Case 1: no sliding

SICOPOLIS (SIA)  Elmer/Ice (FS)  Bamber et al.
Case 2: with sliding

Basal thermal condition (no sliding)
Basal thermal condition (sliding)

SICOPOLIS (SIA) Elmer/Ice (FS)

Relative temperature

Weird!

Basal dynamical conditions (no-sliding)
Basal dynamical conditions (no-sliding)

Conclusions

- Able to overcome limits of SIA:
  \[ \varepsilon = H/L \sim Q \text{ (whatsoever)} \]

- Numerically more intensive

- Open issues:
  - Increase of (local) resolution
  - Horizontal mesh adaption
  - Pre-conditioners, parallel solvers
  - Treatment of energy balance
  - Prognostic runs (high res. climate input?)

THANK YOU!